Comparison of Pricing Approaches for Longevity Markets

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Overview

Background

- Longevity Risk in Pensions and annuity portfolios.

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- LAGIC in Australia ; Solvency II in U.K.

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- LAGIC in Australia ; Solvency II in U.K.
- Longevity-linked securities: Bonds, swaps, options.

Motivation

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- Create longevity instruments allowing for capital markets to buy into.
- Challenge: find "the" fair price for longevity risk.

Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.

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- Mortality modeling and forecasting under the CBD-model with a state-space representation.
- Investigate four approaches used to price S-forwards
- Comment on the results based on the four approaches.

Cairns Blake and Dowd Model

Denote a 1-year death probability for a person currently aged x at time t by $q_{x,t}$, this is modeled via,

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_{1,t} + \kappa_{2,t}(x-\bar{x}). \tag{1}$$

Where $\bar{x}$ is the average of the ages. We adapt this model to incorporate an error component in the measurement equation so that a state-space approach can be applied.

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_{1,t} + \kappa_{2,t}(x-\bar{x}) + \varepsilon_t \tag{2}$$

Cairns et al. (2006) suggests that $\kappa_{1,t}$ and $\kappa_{2,t}$ can be modeled by a 2-dimension random walk with drift,

$$\begin{bmatrix}\kappa_{1,t}\\ \kappa_{2,t}\end{bmatrix} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + \begin{bmatrix}\kappa_{1,t-1}\\ \kappa_{2,t-1}\end{bmatrix} + \omega_t$$

State-space framework

Our framework is as follows:

$$\boldsymbol{y_t} = \begin{bmatrix} ln\left(\frac{q_{x_1,t}}{1-q_{x_1,t}}\right) \\ \vdots \\ ln\left(\frac{q_{x_n,t}}{1-q_{x_n,t}}\right) \end{bmatrix} = \begin{bmatrix} 1 & (x_1-\bar{x}) \\ 1 & (x_2-\bar{x}) \\ \vdots & \vdots \\ 1 & (x_n-\bar{x}) \end{bmatrix} \begin{bmatrix} \kappa_{1,t} & \kappa_{2,t} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon_{x_1,t}} \\ \vdots \\ \boldsymbol{\varepsilon_{x_n,t}} \end{bmatrix}, \tag{3}$$

$$\begin{bmatrix} \kappa_{1,t} \\ \kappa_{2,t} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \kappa_{1,t-1} \\ \kappa_{2,t-1} \end{bmatrix} + \omega_t. \tag{4}$$

Where $\boldsymbol{\varepsilon_t} \sim$ i.i.d $N(0, \sigma_\varepsilon^2)$ and $\omega_t \sim N(0, \Sigma_\omega)$.

Prior Choices

Parameters are estimated Markov Chain Monte Carlo method[1],

$$\pi(\sigma_\varepsilon^2) \sim I.G(a_\varepsilon, b_\varepsilon)$$

$$\pi(\theta) \sim N(\mu_\theta, \Sigma_\theta)$$

$$\pi(\Sigma_\omega|\Sigma_{11}, \Sigma_{22}) \sim I.W\left(\nu + 2 - 1, 2\nu \; diag\left(\frac{1}{\Sigma_{11}}, \frac{1}{\Sigma_{22}}\right)\right)$$

$$\pi(\Sigma_{kk}) \overset{i.i.d}{\sim} I.G\left(\frac{1}{2}, \frac{1}{A_k}\right) \quad \text{for} \quad k \in (1, 2)$$

Priors were chosen such that they had conjugate forms to their respective likelihoods. A hierarchical structure was chosen for Σ_ω, to avoid a biased estimation from a regular Inverse-Wishart prior (Huang et al., 2013; Gelman et al., 2006). The hyper parameters were chosen such that the priors were non-informative.

[1] I.G *Inverse.Gamma*(α, β), N is *Normal*(μ, σ^2), I.W is *Inverse.Wishart*(ν, ϕ)

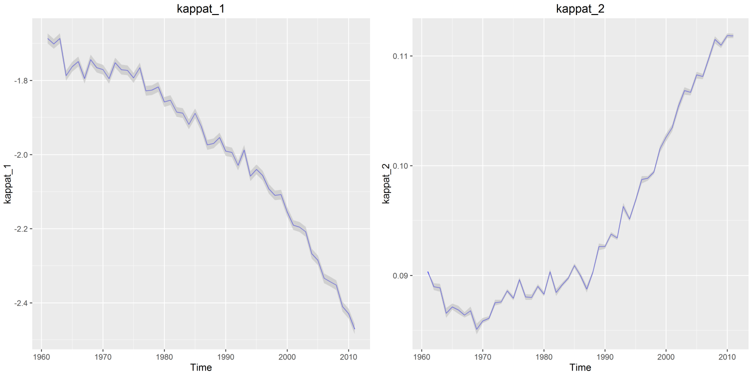
Summary statistics

N=10000 draws, 3000 burn-in period, using joint mortality of Australian dataset 1961-2011 taken from the Human Mortality Database(HMD)

Table 1: Summary Statistics for the estimated parameters

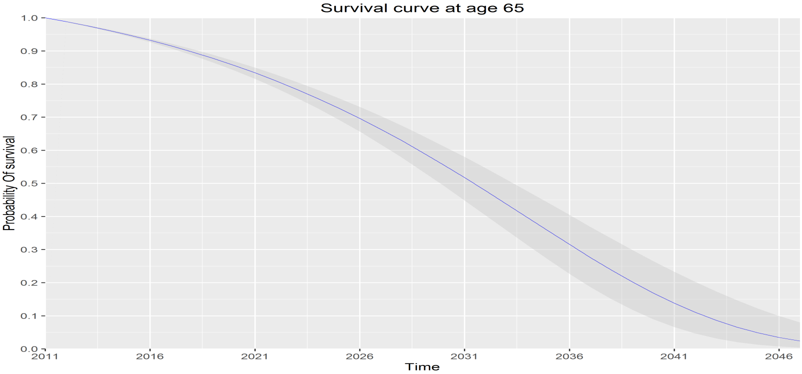
Parameter	Posterior Mean	95% HPD
θ_1	4.79325×10^{-2}	$(-2.48523 \times 10^{-2},\ -5.81384 \times 10^{-2})$
θ_2	4.22871×10^{-3}	$(7.29511 \times 10^{-5},\ 7.69795 \times 10^{-4})$
σ_ε^2	2.63008×10^{-2}	$(2.45841 \times 10^{-3},\ 2.8157 \times 10^{-3})$
Σ_{11}	1.15681×10^{-3}	$(7.11692 \times 10^{-4},\ 1.82368 \times 10^{-3})$
Σ_{12}	2.42572×10^{-5}	$(1.08446 \times 10^{-5},\ 4.249324 \times 10^{-5})$
Σ_{22}	1.56558×10^{-06}	$(9.0352 \times 10^{-7},\ 2.516597 \times 10^{-6})$

Fitted curves κ_t



Forecast of survival curve

Consider the k^th step, Let N be the draws after the burn in period and $n = 1, ..., N$. Then, $\boldsymbol{\kappa}^{(n)}_{T+k} \sim N(\boldsymbol{\kappa}^{(n)}_{T+k-1} + \boldsymbol{\theta}^{n}, (\Sigma_\omega)^{(n)})$, $\boldsymbol{y}^{(n)}_{T+k} \sim N(\kappa^{(n)}_{T+k,1} + (x - \bar{x})\kappa^{(n)}_{T+k,2}, (\sigma^2_\varepsilon)^{(n)}\mathbb{I})$.



We studied at 4 different pricing approaches:

1) Risk-neutral method (Cairns et al., 2006)
2) The 2-factor Wang transform (Wang, 2002)
3) Canonical valuation/ Maximum entropy method (Li and Ng, 2011)
4) An economic approach/ Tatonnement economics (Zhou et al., 2015)

The first two of these methods require data to find the risk-premium λ. Hence, we will use the issued but not sold EIB-bond to calibrate.

EIB bond

Using the setup for the EIB-bond Cairns et al. (2006), we apply Australian mortality projections to males aged 65 with a longevity spread of $\delta = 0.002$ over a $T = 25$ year period.

1) The price obtained by EIB/BNP was in 2004, we assume that the prices have not been inflated since that time for 2011.
2) The original EIB-bond was setup for England and Welsh males aged 65, we assume the same longevity spread δ for Australian population.
3) For ease of calculations, we assume a constant interest rate of 3%.

Let $\bar{\Pi}_t(x, T)$ be the bond price at time t, and $\hat{S}(x, i)$ is the risk-neutral survival probability then,

$$\bar{\Pi}_t(x, T) = \sum_{i=1}^{T} P(t, i) e^{\delta i} \hat{S}(x, i)$$

Under these assumptions, we find that the bond price $\bar{\Pi}_0 \approx 13.46739$

S-forward

Definition

An S-forward contract is a swap where the fixed rate payer pays an amount $K \in (0, 1)$ in exchange for the realised survival probability $_Tp_x$. An S-forward contract written for a population aged x at time t, over a maturity period T, will thus have a pricing formula under risk-neutral density is given by:

$$SF(x, t, T, K) = P(t, T)E_{\mathcal{Q}}\left[{}_Tp_x - K|\mathcal{F}_t\right].$$

Since an S-Forward contract has \$0 inception cost, we have to find the value of $K(T)$ such that the there will be no upfront cost.
$K(T) = E_{\mathcal{Q}}\left[{}_Tp_x|\mathcal{F}_t\right]$

Pricing an S-forward

Under the 4 different pricing methodologies, if we were to price an S-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
$K(T) = E_{\mathcal{Q}}\left[{}_T p_x | \mathcal{F}_t\right] = \tilde{S}(x, T)$

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2) Under Wang Transform Method,
$K(T) = E\left[Q\left(\Phi^{-1}(S(x,t)) + \lambda\right)\right]$

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4) Under the Tatonnement Approach, the value $K(T)$ is determined by the market based on supply and demand.

Risk-neutral pricing method

Our 2-D random walk with drift process:

$$\boldsymbol{\kappa_t} = \boldsymbol{\theta} + \boldsymbol{\kappa_{t-1}} + (\Sigma_\omega)^{\frac{1}{2}} Z$$

Where, $(\Sigma_\omega)^{\frac{1}{2}}(\Sigma_\omega)^{\frac{1}{2}} = \Sigma_\omega$ and $Z \sim N(0, \mathbb{I})$ is under real-world probability measure $\mathbb{P}$. Cairns et al. (2006) suggests that similar to the continuous time case, we can convert to the risk-neutral density (equivalent martingale measure) by,

$$\tilde{Z} = \lambda + Z,$$

Where, λ is the market price of longevity risk and $\tilde{Z} \sim N(0, \mathbb{I})$ under $\mathcal{Q}$, Then,

$$\boldsymbol{\kappa_t} = \boldsymbol{\kappa_{t-1}} + (\boldsymbol{\theta} - (\Sigma_\omega)^{\frac{1}{2}} \lambda) + (\Sigma_\omega)^{\frac{1}{2}} \tilde{Z}$$

Risk-neutral pricing method

Under risk-neutral assumption the EIB-bond price is given by:

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) E_{\mathcal{Q}(\lambda)} \left[e^{-\int_t^i \mu_x(u) du} | \mathcal{F}_t \right]. \tag{5}$$

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i) e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i) \tilde{S}(65, i, \lambda)$$

Market Price of Risk	Value	$\bar{\Pi}_0(65, 25)$
(λ_1, λ_2)	(0.27307, 0.27307)	13.46739
(λ_1, λ_2)	(0.24505, 0)	13.46739

2-factor Wang Transform

Wang (2002) proposes a universal pricing method, such that, assuming we have a liability X over a time period $[0, T]$ with $F_X(x) = P(X < x)$, then with a market price of risk λ, the risk-adjusted (distorted) function of $F(X)$ can be found by,

$$F^*(x) = Q\left(\Phi^{-1}(F(x)) + \lambda\right)$$

Where, $F^*(x)$ is the risk-adjusted function for $F(x)$ and $Q \sim Student - t(\nu)$, Since our aim is to find the risk-neutral survival probability(our underlying), we have,

$$\tilde{S}(x,t) = E\left[Q\left(\Phi^{-1}(S(x,t)) + \lambda\right)\right] \text{ for } t \in [0, T]$$

.

2-factor Wang Transform

To find λ using the EIB-bond

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) Q\left(\Phi^{-1}(S(x, T)) + \lambda\right). \tag{6}$$

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i) e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i) \tilde{S}(65, T)$$

Market Price of Risk	Value	$\bar{\Pi}_0(65, 25)$
λ	0.3478043	13.46739

Canonical Valuation

The maximum entropy principle was first proposed by Stutzer (1996) and used by Kogure and Kurachi (2010); Foster and Whiteman (2006) used in longevity context to find market survival probability denoted by ${}_Tp_x^{\text{market}}$. In our case we by using the EIB-Bond in combination with the maximum entropy principle to find ${}_Tp_x^{\text{market}}$.

Canonical Valuation methodology

1. Let $\boldsymbol{p}_{\boldsymbol{x}}^{j} = ({}_1p_x^j,\ {}_2p_x^j, \dots,\ {}_Tp_x^j)$, for $j = 1, \dots, N$, and let $\boldsymbol{\pi}$ denote the empirical distribution for $\boldsymbol{p}_{\boldsymbol{x}}$
2. $\bar{\Pi}$ denotes the market price of the EIB-bond $\bar{\Pi}(65, 25)$.
3. Let $\boldsymbol{\pi}^*$ be the risk-neutral distribution for $\boldsymbol{\pi}$, then $\sum_{j=1}^{N} \Pi^j \pi_j^* = \bar{\Pi}$.
4. Then the maximum entropy principle stipulates that, $\boldsymbol{\pi}^*$ should minimize the Kullback-Leiber Information divergence, $\sum_{j=1}^{N} \pi_j^* \ln\left(\frac{\pi_j^*}{\pi_j}\right)$, subject to the constraint $\pi_j^* > 0$ and $\sum_{j=1}^{N} \pi_j^* = 1$.

Canonical Valuation methodology

1. Kapur and Kesavan (1992) derived the solution to the minimization of $\sum_{j=1}^{N} \pi_j^* \ln\left(\frac{\pi_j^*}{\pi_j}\right)$ subject to $\bar{\Pi}$, which is given by $\hat{\pi}_j^* = \frac{\pi_j \exp\left(\gamma \bar{\Pi}^j\right)}{\sum\limits_{j=1}^{N} \pi_j \exp\left(\gamma \bar{\Pi}^j\right)}$
2. Find γ from, $\bar{\Pi} = \frac{\sum\limits_{j=1}^{N} \bar{\Pi}^j \exp\left(\gamma \bar{\Pi}^j\right)}{\sum\limits_{j=1}^{N} \exp\left(\gamma \bar{\Pi}^j\right)}$.
3. $\sum\limits_{j=1}^{N} {}_t p_x^j \pi_j^* = {}_t p_x^{\text{market}}$

In our case, we don't assume there is a risk premium λ, but there is a γ parameter which "corrects" the real world probability ${}_t p_x$to adjust for the market accepted ${}_T p_x^{market}$.

Tatonnement Approach

This approach was first suggested by (Zhou et al., 2015). We price our S-forward instrument based on the equilibrium price that matches market supply and demand.

1. Assume we have a buyer (investor (B)) of an S-forward and a seller (hedger (A)).

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3. Then,
   $\theta_A = \sup_{\theta_A} E\left[U\{\omega_{t-1}^A e^r - \theta^A g(S(x,t)) - f(S(x,t))\}\right]$

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4. $\theta_B = \sup_{\theta_A} E\left[U\{\omega_{t-1}^B e^r + \theta^B g(S(x,t))\}\right]$

Tatonnement Approach

Since g is an S-forward,we have, $g = (S - K)$. Choosing an Exponential Utility function, the following algorithm is used to obtain a price K. (Zhou et al., 2015).
for each time period $t \in [1, T]$

1. Guess an initial K.

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2. Determine θ_A and θ_B.

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4. else, update K by:
    1) $K^{i+1} = K^i + h^i$
    2) Where $h^i = \gamma |K^i| (\theta_B - \theta_A)$

Results

For each of the methods, 1000 samples from the MCMC was used after burn-in. A Monte-Carlo average was taken when an expectation was involved. Using a portfolio which consists of people aged 65 at time 0, with a hedging period of $T = 5, 10, 15, 20, 25$ of the S-forward. The prices are shown below:

Period	real-world	Risk-Neutral	Wang-T	Canonical	Tat
$K(5)$	0.93233	0.93577	0.96720	0.93219	0.93225
$K(10)$	0.83398	0.84430	0.90622	0.83378	0.83398
$K(15)$	0.69621	0.72295	0.80547	0.69648	0.69621
$K(20)$	0.51740	0.56046	0.65226	0.52053	0.51740
$K(25)$	0.31560	0.37075	0.44740	0.32598	0.31790

Comments

- Bayesian inference allows us to have prediction uncertainty in a systematic way via the prior distribution.
- The different choices of pricing approaches, produced "similar" results, except for the tatonnement approach. Under economic conditions, it shows that there really isn't a need for a "premium" if both investor and hedger acts "rationally".
- Under the transformation method, the premium is much higher than other approaches. This is because the effect of the distortion operator causes a greater change in mortality directly compared with the risk-neutral method.

Future research

- Investigation is still going on, to finding the "correct' way to price instruments.
- At the moment this is all theoretical work, this paper introduces these methods, and applies to pricing an S-forward contract.
- Next ...

Thanks for Listening

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Markov-Chain Monte-Carlo (MCMC)

Let $\psi = (\sigma_\varepsilon^2, \Sigma_\omega, \boldsymbol{\theta})$. An MCMC method will be used to explore the posterior distribution and parameter states.

- Obtain initial draws denoted by ψ^0.
- Conditional on ψ^0, find the distribution of latent states via the Kalman Filter.
- latent variable $\kappa_{1:T}$ drawn recursively from Backward Sampling (Carter and Kohn, 1994).
- Conditional on drawn latent variable, draw model parameters from their respective conditional posterior density.

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