

Pricing an inflation-linked annuity considering interest rate and longevity risk in the HJM framework

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Abstract

Interest rate risk, longevity risk and inflation risk have been found in an annuity market. However, there is no research dealing with these three risk factors simultaneously for pricing annuity products. Moreover, longevity risk may affect the inflation risk because the increase of life expectancy may cause greater inflation risk. Therefore, when dealing with inflation risk for inflation-linked annuity products, we can't ignore the longevity risk. This paper first provides an annuity model which combines inflation risk, interest rate and longevity risk. We assume the dynamics of interest rate term structure and inflation rate are normally distributed under Heath-Jarrow-Morton framework (HJM, 1992) and the mortality rate follows a logarithm normal distribution. As a result, we obtain the closed form pricing solution

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for the exotic structures of inflation-linked annuities.

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Introduction

After financial crisis in 2008, it has highlighted the sensitivity and vulnerability of financial markets to inflation, which reduces the value of money and affects the net returns of financial instruments. In response to this, investors who are concerned with maintaining their investment's purchasing power rather than its market value are resorting to inflation linked (IL) products to hedge their inflation risk. Consequently, the inflation market has been rapidly growing for the last decade and has further great potential growth worldwide. As for in the annuity market, the increase of life expectancy has addressed the inflation risk as an important issue in order to ensure that future consumption will not fall below a minimum acceptance standard of living because of longer horizons. According to a retirement survey by the Society of Actuaries (2011), 71% of preretirees and 58% of retirees in the US are concerned with the ability to keep the value of their savings and investments up with rising inflation.

Besides inflation risk, annuity providers may incur higher than expected benefit costs owing to the longer life expectancy, called longevity risk. Life annuities are an important product that allows individuals to manage longevity risk in retirement. Annuities involve the transfer of longevity risk from annuitants to the life insurer.

During the last decade, longevity risk has become increasingly intensive for annuity providers to manage. In 2007, these institutions' exposure to improvements in life expectancy amounted to a staggering 400 billion USD in the UK and the US alone. In the UK, a pension plan buyout market started in 2006 to transfer corporate pension plan assets and liabilities to insurance companies. The buyout market transfers interest rate and inflation risks and not just longevity risk. However, Bodie (2003) points out that investment and traditional annuity products in most of the retirement portfolios are not protected against inflation. Moreover, Brown et al. (2001) present evidence that the inflation risk hedging property of US stocks and long-term bonds is limited. Recently, the majority of the annuity studies have been on either longevity risk or interest rate risk. However, in addition to longevity risk, and interest rate risk, inflation risk exists in the same context. Tiong (2013) presents various inflation-linked variable annuities which are designed to help investors protect their portfolios from inflation risk. This paper focuses on the inflation rate dynamics and obtains closed-form pricing formulas for these inflation-linked annuity products. However, interest rate, inflation rate and mortality rate are three major factors regarding the inflation-linked annuity product. To fill this gap, we attempt to deal with the inflation-linked annuity which involves interest rate, inflation and longevity risk.

Under Heath-Jarrow-Morton (HJM) type of framework where interest rate term

structures are assumed to be normally distributed, the closed form pricing results can be obtained. Additionally, this model can be reduced to Tiong (2013) as longevity risk does not exist. To the best of our knowledge, this paper is the first one to deal with the life annuity product with inflation risk, interest rate risk and longevity risk.

This paper is structured as follows. Section 2 outlines an economy for an inflation-protected annuity with longevity risk, where the interest rate structure follows a HJM model, and the mortality dynamic follows a process distributed in a logarithm normality. Section 3 presents the payout design of inflation-linked annuity product and its analytical pricing formula, as well as the sensitivity analysis. Finally, Section 4 concludes the paper.

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