Mortality collar

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LONGEVITY EIGHT

Eighth International Longevity Risk and Capital Markets Solutions Conference

WATERLOO University SEPTEMBER 7 & 8 2012

1er septembre 2012

Programme

General context

Motivation

- Some derivatives exist for hedging against mortality or longevity risks (Bauer, Börger & Rub 2008, Cairns, Blake & Dowd 2006, Cox, Lin & Milidonis n.d.),
- Some of them are on natural hedging (Cox & Lin 2004).

We develop here a contract based on natural hedging of downside risks transfer mechanism between a life insurer portfolio and an annuity's one with the mortality rate quantiles as bounds for a cap and a floor.

The collar principle

a cap and a floor

At each constatation date :

- if $q_{x,t} > q^{\beta}_{x,t}$ then the losses of the life insurer corresponding to the exceptional rising of the mortality from $q^{\beta}_{x,t}$ to the observed $q_{x,t}$ are compensated by the pension fund ;
- similarly, if $q_{x,t} < q^{\alpha}_{x,t}$ then the losses of the pension fund are compensated by the life insurer ;
- at least if the mortality rate $q_{x,t}$ lie in the collar no one have to pay to the other.

The data

We consider French mortality data ([1]).

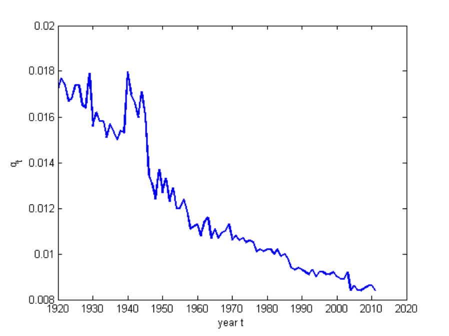

FIGURE: Historical mortality rate of French population (1920 to 2011)

The mortality model

(Lee & Carter 1992) and (Cox et al. n.d.)

$$q_{x,t} = e^{\alpha_x + \beta_x \kappa_t + \epsilon_{x,t}} \qquad (1)$$

The mortality model

(Lee & Carter 1992) and (Cox et al. n.d.)

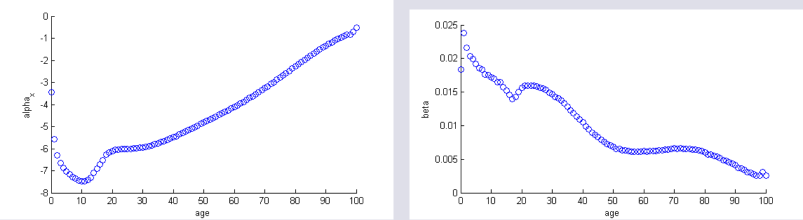

FIGURE: Estimated values of α_x & β_x

The mortality model

(Lee & Carter 1992) and (Cox et al. n.d.)

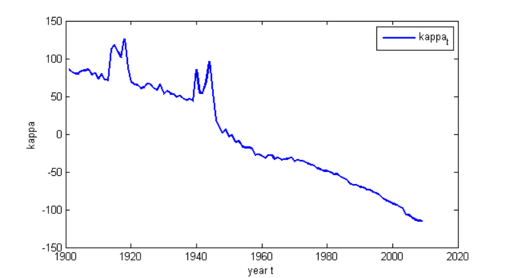

FIGURE: Estimated values of κ_t

The mortality model

(Lee & Carter 1992) and (Cox et al. n.d.)

TABLE: Statistics of error term from model 4

Parameter	Parameter value
Mean	7.10^{-6}
Standard deviation	10.5973
Kurtosis	10.8995
Skewness	0.1910

The kurtosis of υ_t 10.8995 ans its skewness 0.1910.

The mortality model

TABLE: Calibrated parameters of Regime-switching lognormal and lognormal models

Parameters	LN model	RSLN model
log-likelihood	-411.4676568	-0.010717659
μ	$-3.66972.10^{-6}$	
σ	10.54861	
μ_1		0.032898375
μ_2		-0.144669812
σ_1		8.504312799
σ_2		8.229352177
p_{12}		0.9999
p_{21}		$9.3982.10^{-6}$

The risk measure

Utility function (Gerber & Pafumi 1998)

HARA-CRRA power utility function of the second kind :
$u(x) = \frac{x^{1-c}-1}{1-c}$ (with $c > 0$).

$$E_Q[V(X)] = \frac{E[V(X)\,u'(X)]}{E[u'(X)]} \tag{2}$$

With the choice of $X = X_t(s) = q_{x_t,s}$ we have

$$E_Q\left[V\left(q_{x_t,s}\right)\right] = \frac{E\left[V\left(q_{x_t,s}\right) {q_{x_t,s}}^{-c}\right]}{E\left[{q_{x_t,s}}^{-c}\right]} \tag{3}$$

.

The life insurer's over quantile loss

Projected liabilities differences

$$E_Q\left[V_1\left(q_{x_t,s}\right)\right] = E_Q\left\{\left[LF\left(x_t,s\right) - VaR_\beta\left[LF\left(x_t,s\right)\right]\right]1_{\left[LF>LF_\beta\right]}\right\} \quad (4)$$

.

The pension fund over quantile loss

Projected liabilities differences

$$E_{Q_t}\left[V_2\left(q_{x_t,s}\right)\right] = E_{Q_t}\left\{\left[FP\left(x_t,s\right) - FP_\alpha\left(x_t,s\right)\right] 1_{[LF>LF_\alpha]}\right\} \qquad (5)$$

.

The pension fund over quantile loss

Projected liabilities differences

$$E_{Q_t}\left[V_2\left(q_{x_t,s}\right)\right] = \frac{P\,N_{x_t}\,e^{Ri_{x_t}\,t}}{1-e^{-Ri_{x_t}}}\left\{e^{-Ri_{x_t}\,T^{\alpha}_{x_t}}\,\frac{E_t\left[q^{-c}_{x_t,s}\,1_{[q<q^{\alpha}]}\right]}{E_t\left[q^{-c}_{x_t,s}\right]} - \frac{E_t\left[e^{-Ri_{x_t}\,T_{x_t}}\,q^{-c}_{x_t,s}\,1_{[q<q^{\alpha}]}\right]}{E_t\left[q^{-c}_{x_t,s}\right]}\right\} \quad (6)$$

.

Duality

Feasibility of the contract

The feasibility of the contract is conditioned by the fact that at the present time each party has the same expected loss :

$$E_{Q_t}\left[V_1\left(q_{x_t,s}\right)\right] = E_{Q_t}\left[V_2\left(q_{x_t,s}\right)\right] \quad (7)$$

. From this feasibility equation and with evident notations we get a relation between α and β :

$$\frac{C}{P}\left(1-e^{-Ri}\right)e^{Rs}\left\{E_t\left[q^{1-c}\,1_{[q>q^\beta]}\right]-q^\beta\,E_t\left[q^{-c}\,1_{[q>q^\beta]}\right]\right\}$$
$$=\left\{e^{-Ri\,T^\alpha}\,E_t\left[q^{-c}\,1_{[q<q^\alpha]}\right]-E_t\left[e^{-RiT}\,q^{-c}\,1_{[q<q^\alpha]}\right]\right\} \quad (8)$$

.

Duality

The method

If β is known, we have to derive the right difference without knowing α numerically by the least square of the two differences $(\sqrt{\left(E_{Q_t}\left[V_1\left(q_{x_t,s}\right)\right] - E_{Q_t}\left[V_2\left(q_{x_t,s}\right)\right]\right)^2})$.

Conclusion

Remarks The solutions are functions

1. of the ratio of C and P to rebalance the payment if one counterpart pays more than the other,
2. but not of the length of the population, instead of the lengths are not the same.

The market data are the European interbank and zero-coupon bonds rates for the term structure and the inflation rates are from the Agence France Trésor consumer price index without tobacco.

Conclusion

Discussion : work in progress...

1. The calibration of the CRRA coefficient is more "complex" than expected...,
2. simulations results, efficacity tests and continuous times interest and inflation rates models are coming soon.
3. since this contract deals with downside risks, what about extreme values theory ?

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