Life Settlement Pricing

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Life Settlement Description

- A life settlement is a financial arrangement whereby the third party (or investor) purchases a life insurance policy from the person who originally purchased a life insurance policy.

- This third party pays the insured an amount greater than the cash surrender value of the policy -- in effect, the trade-in value of the policy as determined by the originating insurance company-- but less than the face value (or the death benefit).

- It can be a win-win situation, as the investor can obtain a return on their initial investment and premium payments once the death benefit becomes payable (assuming the insured does not live too much longer than expected when setting the purchase price) and the owner of the policy obtains more money than they otherwise could obtain by surrendering their policy.

- It is estimated that in the past five years more than $40 billion of face value has been sold in the life settlement market and the market size will grow from $13 billion in 2004 to $161 billion over the next few decades

Brief History

- Longevity risk traditionally viewed though its impact on pensions, social security systems and corporate defined benefit plan solvency, but the life settlement market is even more vulnerable to longevity risk.
- Illustrative of this is what happened to the viatical settlement market, the precursor of life settlement market.
- AIDS patients sold policies. Profitable to investors, until 1996 when papers were presented at the International AIDS Conference in Vancouver, that gave evidence of a new drug capable of substantially reducing, perhaps even to zero, the level of HIV in its infectees.

Brief History

- The effect became evident in the collapsed value of the viatical settlement firms, (e.g., Dignity Partner), and the significant decrease in prices offered to AIDS victims for their insurance policies; with evidence that policies might take a substantially longer time to mature, their value plummeted.
- As market for viatical settlements plunged, companies, expanded life insurance purchases to those belonging to the elderly. Elderly with estimated low life expectancies chosen because a low life expectancy meant a greater possibility of profiting early. Today, this life settlement market has increasing potential as baby boomers are enter old age
- To create distance between the association with AIDS and the conceivably negative connotation that the term “viatical settlement” enlisted, companies chose the different title, “life settlement.”

Life settlement Pricing

- The main factor in the life settlement securities pricing currently is the estimation of the life expectancy of the insured (and the premium payments), but other information may also be available.

- The life expectancy of the insured at the time of sale (settlement) is often considered in the pricing as the major random variable which influences the sales price to the insured when he sells his life insurance policy to the third party as a life settlement.

Pricing Issues

The net present value of future payment of the life settlement product is contingent on the future life time T of the insured.

Jensen's inequality says for any convex function f and any random variable Z, $E[f(Z)] \geq f(E[Z])$.

Thus, since v^T is convex in T, according to Jensen's inequality, using the expected life time E[T] only and pricing the product like one would an E[T]-year bond with a pay off of $v^{E[T]}$, v being the discount rate, results in incorrect pricing. This price $v^{E[T]}$ is always smaller than the value of the $E[v^T]$ which is the true expected net present value. Thus treating the life settlement as a bond of duration E[T] is underprices the value of the payoff

Pricing Issues

This carries over into the time zero price of the life settlement. If P is the premium to be paid at the beginning of each year then the buyer pays an amount equal to

$$P[\frac{1-v^T}{iv}]$$

The time zero value of the life settlement, X is a convex function of T, the future lifetime since v^T is convex and

$$X(T) = Bv^T - P[\frac{1-v^T}{iv}] = v^T[B+\frac{P}{iv}]-[\frac{P}{iv}]$$

Again according to Jensen's inequality, using the expected life time E[T] only and pricing the product as X(E[T]) instead of E[X(T)] results in systematically underpaying the insured.

Pricing Issues

- **Moral: We should use the entire probability distribution for pricing the life settlement, not just the expected lifetime of the insured.**
- **Problem**: While E[T] can be assessed by a medical underwriter, usually the probability distribution of T is not known, cannot be assessed effectively by the life underwriter, and reasonable candidate distributions (e.g., impaired life mortality tables) for T will not have the specified value of E[T] consistent with the medical underwriter's assessment.
- We may also have other information (e.g., variance, or survival probability for a specified period, or relative mortality risk statistics from the medical literature), and this should be incorporated
- The medical underwriter may be specifying median instead of mean, or likelihood of death within a window of years may be specified

Approaches to Modeling

- We want to use a "standard" or pre-specified distribution for T, but these will not be consistent with medical underwriter or other information. How do we adjust the standard to reflect this known information?
- 1. Could use "accelerated failure time model" from reliability theory. If $S_0(t)$ is the standard survival function, then use $S_\mu(t) = S_0(e^\beta t)$. Speed up or slow down progression along a given survival curve, pick β to give the desired mean μ. Essentially constantly multiplying hazard function to get desired result
- 2. Also proportional hazards model with parametric distribution.
- Also could use an extension of Lee Carter Mortality and adjust it to reflect known information

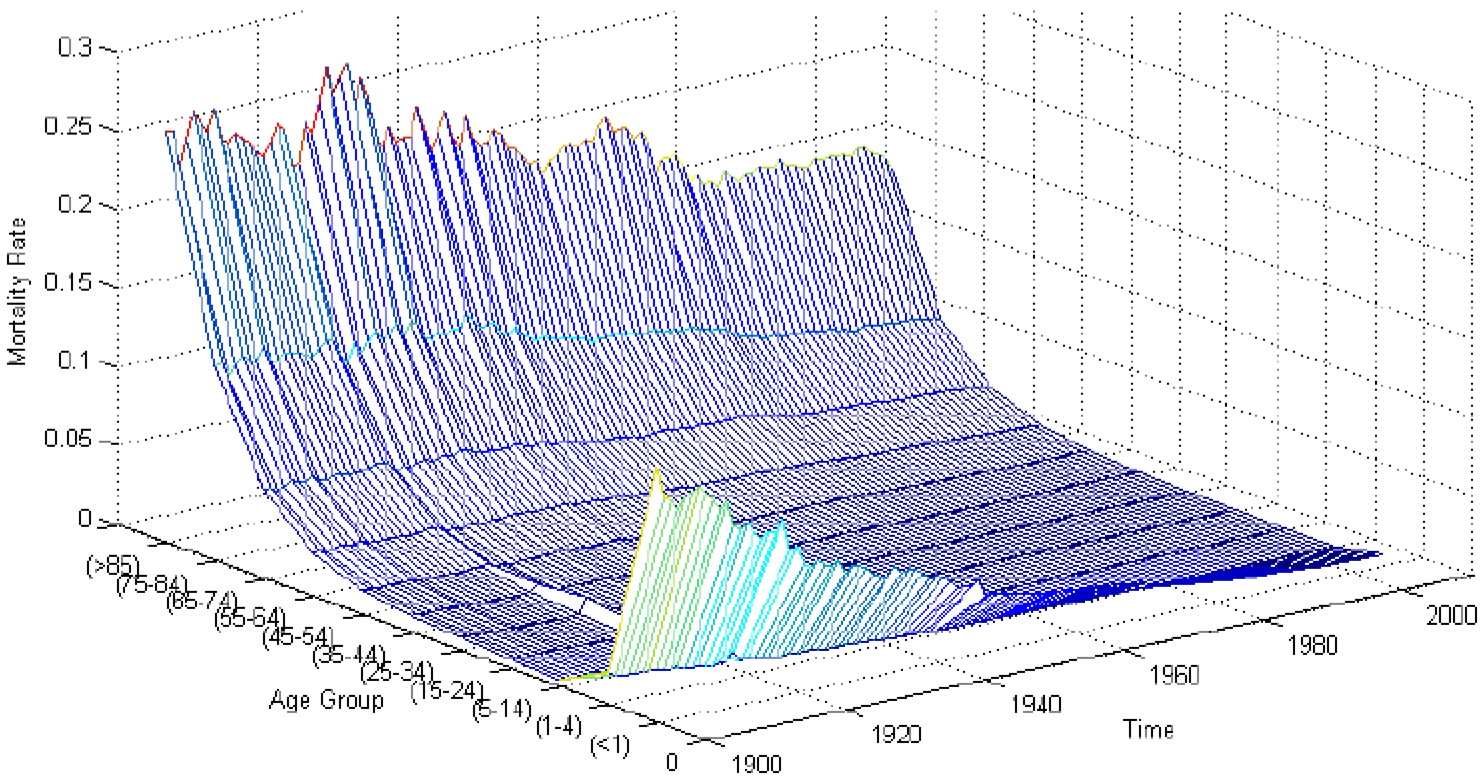
An Information Theoretic Approach

- We propose to start with a chosen standard distribution for T, and then adjust it to get a distribution consistent with the information we have. The statistical adjustment methodology we propose is based upon information theory. It will allow us to adjust any mortality table to obtain exactly some known individual characteristics, while obtaining a table that is as close as possible to the starting standard one.

- In this way, the method provides more accurate projection and evaluation for the life settlement products, through incorporating more statistical information of the insured's future life time. One can then price life settlements using the whole life distribution rather than just life expectations

Example mortality series

Figure 1900-2004 Historical Mortality Rates



Data

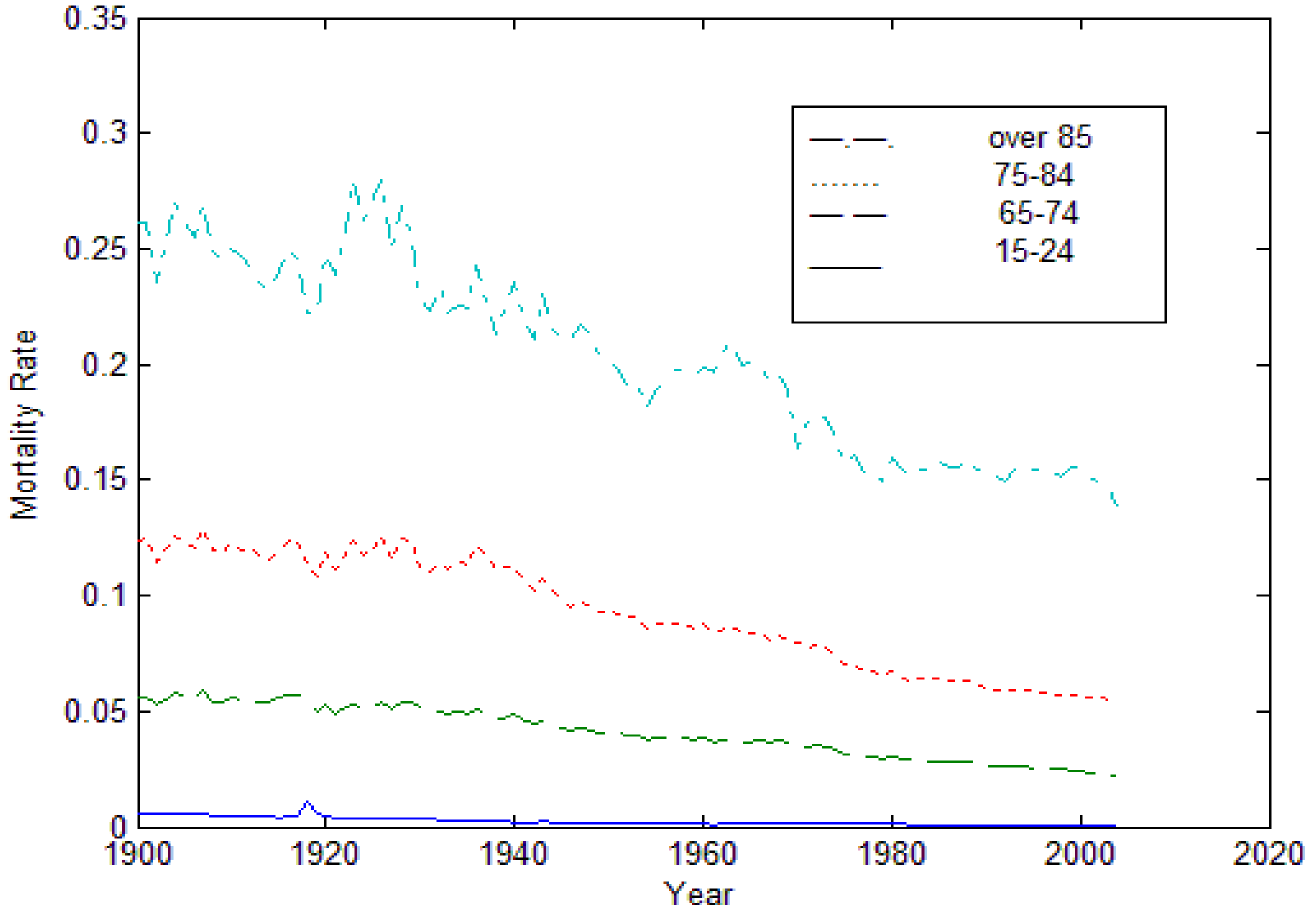


Figure 2. Comparison of the Age Group Mortality Rates

Advantages of model extending Lee Carter as “standard” mortality model

- One model we propose to take as our standard mortality table incorporates the effect of the dynamic longevity risk through the original life table which is generated from the Double Exponential Jump Diffusion model (DEJD) extension of the Lee Carter Model

- The DEJD model incorporates the potential for longevity jump (caused by medical improvement, etc), mortality jump (caused by pandemic influenza, etc) as well as a dynamic main trend of the mortality rate, which provide better explanation and fitness to the historical mortality rate data.

Lee-Carter Mortality Model for Mortality as a Function of Age and Year

- $\ln \mu_{x,t} = \alpha_x + \beta_x * k_t + \varepsilon_{x,t}$ with $\mu_{x,t}$ = crude death rate at age x in year t
- $q_{x,t}$ = mortality rate, age x, year t, = $1 – \exp[- \mu_{x,t}]$
- They assume aggregate mortality as modeled by k_t follows a random walk with drift
- We can estimate for $\{\alpha_x, \beta_x, k_t\}$ for a given cohort
  - $\{\alpha_x, \beta_x\}$ estimated using Singular Value Decomposition and Regression
  - k_t estimated using Maximum Likelihood once $\{\alpha_x, \beta_x\}$ are estimated

Our Model Specification for the Systematic Part

Specification

$$dk_t = \alpha dt + \sigma dW_t + d\left(\sum_{i=1}^{N(t)} (V_i - 1)\right) \qquad (5)$$

$Y = \log(V)$ and

$$f_Y(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y < 1\}} \qquad (6)$$

where $\eta_1, \eta_2 > 0, \quad p, q \geq 0, \quad p + q = 1.$

W_t : standard Brownian Motion

$N(t)$: Poisson process with rate λ

λ : frequency of the jumps

p : proportion of positive jumps

q : proportion of negative jumps

η_1 : scale of positive jumps

η_2 : scale of negative jumps

Features

- Differentiating positive jumps and negative jumps
- Mathematical tractability
- Closed-form formula
- Concise
- Widely implemented

Using this Model Specification We Obtain the Expected Future Mortality

The closed - form expression for the expected future mortality rate $\mu_{x,t}$ is

$$E^*[\mu_{x,t}] = \exp(a_x) \times E^*[\exp(b_x k_t)]$$

$$= \exp(a_x + b_x k_0 + b_x t(\alpha^* - \frac{1}{2}\sigma^2 - \lambda^* \gamma^*) + \frac{1}{2} b_x^2 \sigma^2 t + \lambda^* t(\frac{p\eta_1^*}{\eta_1^* - 1} + \frac{q\eta_2^*}{\eta_2^* + 1} - 1))$$

Model Framework: the Function k_t

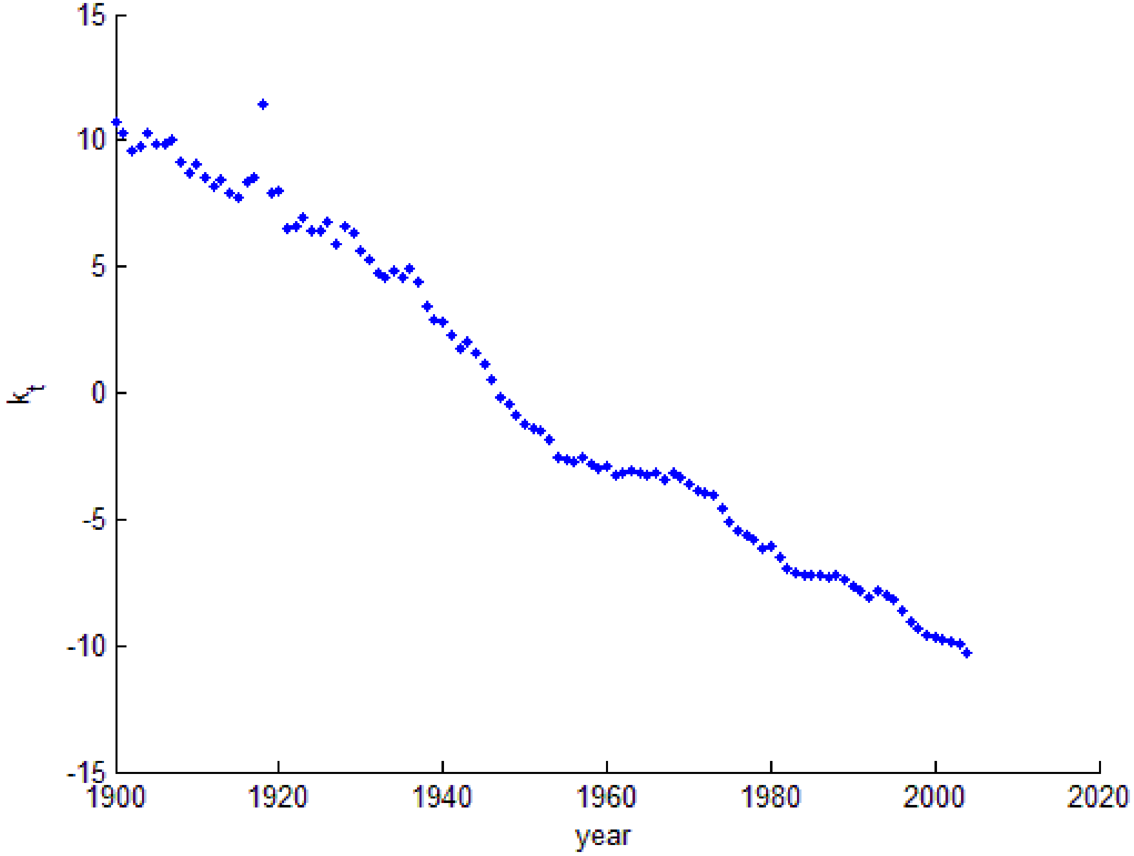


Figure 3. The mortality time - series k_t

Model Comparison

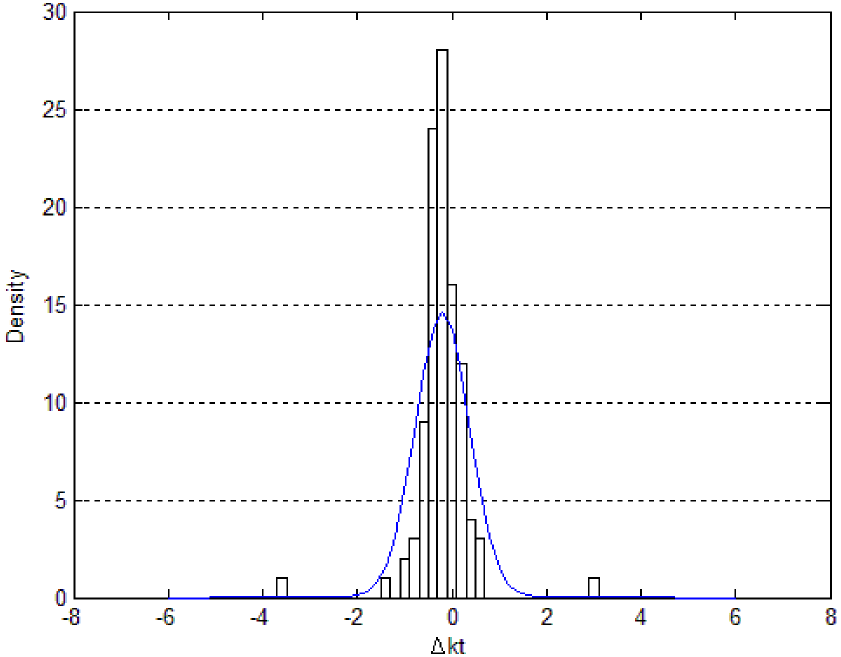


Figure 4. Comparison of Actual Δk_t Distribution and Normal Distribution

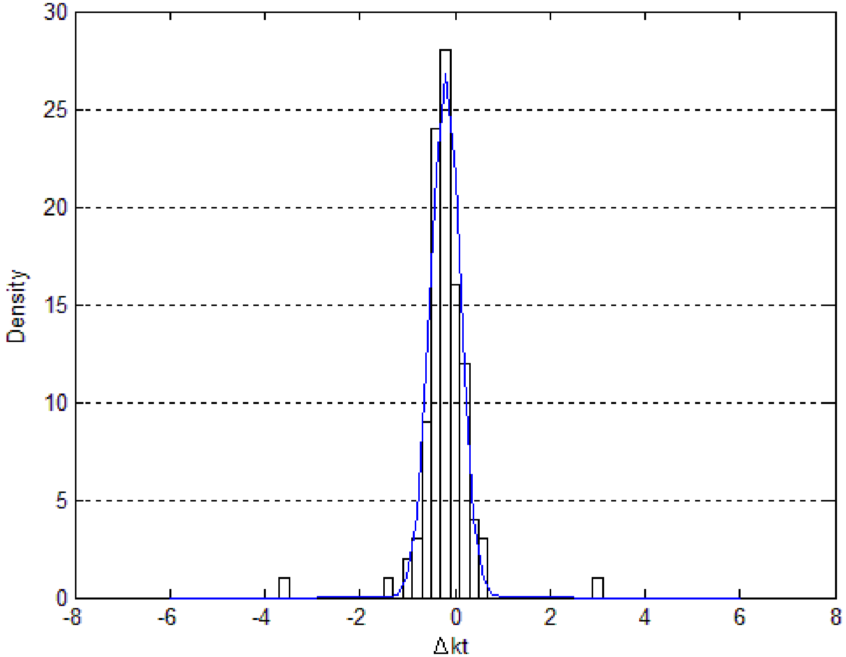


Figure 5. Comparison of Actual Δk_t Distribution and DEJD Distribution

Mean of Normal Distribution is :

$\mu_{BM} = -0.20$

Standard Deviation of Normal Distribution is :

$\sigma_{BM} = 0.57$

Mean of DEJD Distribution is :

$\mu_{DEJD} = -0.20$

Standard Deviation of DEJD Distribution is :

$\sigma_{DEJD} = 0.31$

Model Comparison

- Compare fitness of DEJD model with Lee-Carter Brownian Motion model and Normal Jump Diffusion model (Chen and Cox, 2009)

- Bayesian Information Criterion (BIC)
  - Allow comparison of more than two models
  - Do not require alternative to be nested
  - Conservative, heavily penalize over parameterization
  - The smaller BIC, the better fitness

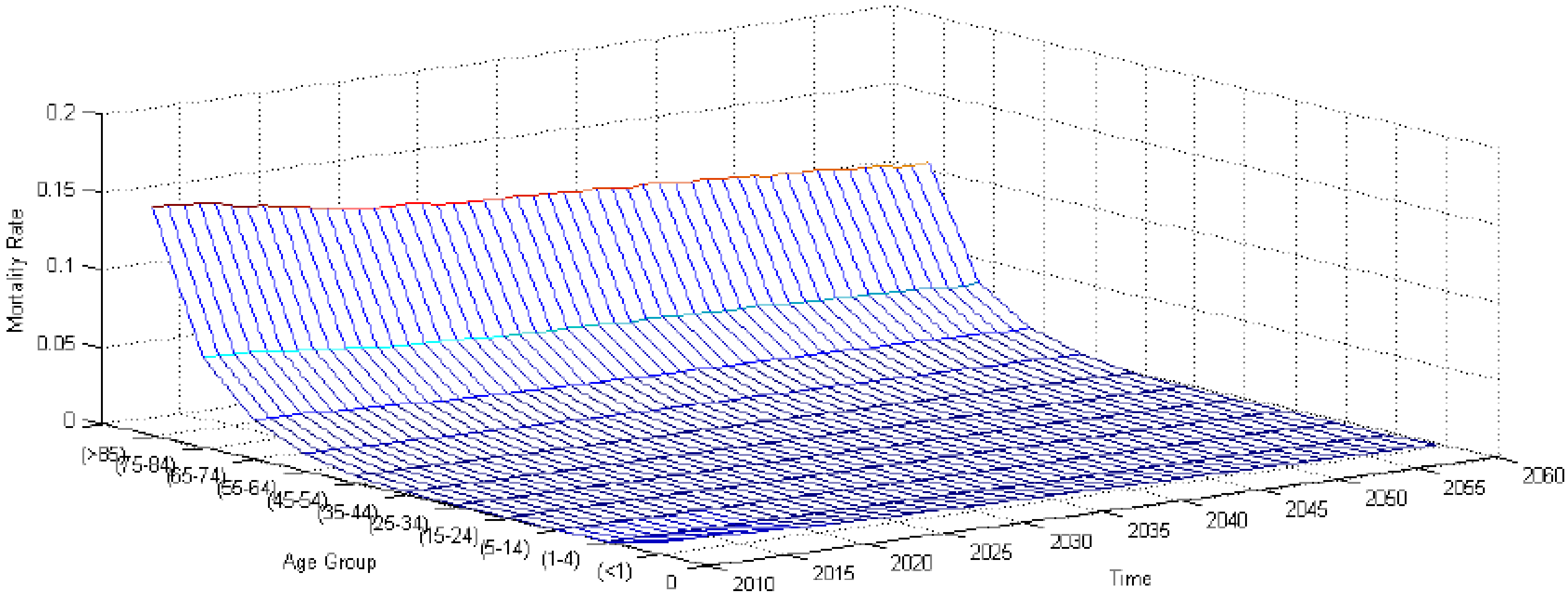
$$BIC_k = -2\ln f(C \mid \theta'_k, M_k) + n_k \ln(m) \qquad (21)$$

	Number of Parameters	ln(likelihood)	BIC
Double Exponential Jump Diffusion Model	6	-49.95	127.76
Normal Jump Diffusion Model	5	-62.52	148.26
Without Jump Diffusion Model	2	-94.27	197.83

Table 2. Comparison of model fitness

Using this model we can also project mortality forward

Figure 12. 2010-2060 Projected Mortality Rates



Information Theory Approach to Getting a Mortality Model for Pricing

For distinguishing between two densities on the basis of an observation t, a sufficient statistic is the log odds ratio in favor of the observation having come from in favor of g. It is the amount of information contained in a observation t for discriminating in favor of f over g. In a long sequence of observations from , the long-run average or expected log odds ratio in favor of f is I(f|g) =

$$E_t = \left(ln \frac{f(t)}{g(t)} \right) = \sum_l f(t_l)\, ln \frac{f(t_l)}{g(t_l)}$$

This reflects the <u>expected</u> amount of information for discriminating between f and g. Note that I(f|g) ≧0 and = 0 if and only if f=g . Thus, the size of is a measure of the closeness of the densities f and g.

For a given g, one can minimize I(f|g) over f to find the closest f. If we have constraints, we can do a constrained optimization. E.g., if the mean is give as m, then we have constraints:

$$1 = \sum f_l, \qquad m = \sum k f_k \qquad (28)$$

We do this with the distribution of life given by the mortality table g and the expectation of life m as given by a life settlement medical expert or actuary to find a best fitting mortality table for pricing this individual's life settlement. We can incorporate almost any expected constraint

Information Theory

- To phrase the problem mathematically, we desire to find a vector of probabilities that solves the problem:

$$\min \mathrm{I}(f|g)$$

- Here is the vector of probabilities corresponding to the standard probability distribution. Brockett, Charnes and Cooper (1980) show that the problem has a unique solution, which is:

$$f_l = g_l \exp\left[-(\beta_0 + 1) - \beta_1 a_1(t_l) - \cdots - \beta_k a_k(t_l)\right]$$

- The parameters can be obtained easily as the dual variables in an unconstrained convex programming problem:

$$\min_\beta \sum \{ g_l \exp[-(\beta_0 + 1) - \beta_1 a_1(t_l) - \cdots - \beta_k a_k(t_l)] - (\beta_0 + \theta_1 \beta_1 + \cdots + \theta_k \beta_k)\}$$

An Example (Life Policy from State Farm)

Table 12. Schedule of Benefits

Form	Description	Initial Amount	Benefit Period Ends	Annual Premium	Premiums Payable
07000	Basic Plan	$50,000	With Life	$565.50	To 2047

Table 13. Parts of Schedule of Insurance and Values

Insurance Amount		Guaranteed Values	
On Insured	May 20,	End of Policy Year	Cash Value Dollars
50,000	1996	Age 60	3,100.00
50,000	1998	Age 62	4,038.50
50,000	2001	Age 65	5,567.00
50,000	2006	Age 70	8,438.50

Table 3. Adjusted and Standard Mortality Table for Age 70
Standard table is the fit DEJD cohort age 70, adjusted to have life expectancy of two years

Year k	Time k'	Age x	Standard Table Rate q_x	Standard Table Probability g_k	Standard Table Survival Function l_k	Adjusted Table Rate q_x'	Adjusted Table Probability f_k	Adjusted Table Survival Function l_k'
2006	0	70	0.02539	0.02539	1.00000	0.30926	0.30926	1.00000
2007	1	71	0.02664	0.02596	0.97461	0.30696	0.21203	0.69074
2008	2	72	0.02796	0.02652	0.94865	0.30328	0.14518	0.47871
2009	3	73	0.02934	0.02706	0.92213	0.29769	0.09929	0.33353
2010	4	74	0.0308	0.02757	0.89507	0.28948	0.06781	0.23424
2011	5	75	0.03233	0.02805	0.8675	0.27785	0.04624	0.16643
2012	6	76	0.03394	0.02849	0.83945	0.26199	0.03149	0.12019
2013	7	77	0.03563	0.0289	0.81096	0.24135	0.02141	0.08870
2014	8	78	0.03741	0.02926	0.78206	0.21592	0.01453	0.06729
2015	9	79	0.03928	0.02957	0.7528	0.18657	0.00984	0.05276
2016	10	80	0.04125	0.02983	0.72323	0.15510	0.00666	0.04292
2017	11	81	0.04385	0.03042	0.69343	0.12540	0.00455	0.03626
2018	12	82	0.04663	0.03092	0.66299	0.09773	0.00310	0.03172
2019	13	83	0.04962	0.03136	0.63208	0.07365	0.00211	0.02862
2020	14	84	0.05282	0.03173	0.60072	0.05392	0.00143	0.02651
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
2038	32	102	0.1774	0.01641	0.09253	0.00000	0.00000	0.00000
2039	33	103	0.19067	0.01451	0.07611	0.00000	0.00000	0.00000
2040	34	104	0.20503	0.01263	0.0616	0.00000	0.00000	0.00000
2041	35	105	0.22058	0.0108	0.04897	0.00000	0.00000	0.00000
2042	36	106	0.23743	0.00906	0.03817	0.00000	0.00000	0.00000
2043	37	107	0.25571	0.00744	0.02911	0.00000	0.00000	0.00000
2044	38	108	0.27552	0.00597	0.02166	0.00000	0.00000	0.00000
2045	39	109	0.29703	0.00466	0.01569	0.00000	0.00000	0.00000
2046	40	110	0.31926	0.00352	0.01103	0.00000	0.00000	0.00000
2047	41	111	1.00000	0.00751	0.00751	0.00000	0.00000	0.00000
Total				1.00000			1.00000	

Life Settlement Structure

Basic Definitions:

B : Net death benefit (or face value) of the life policy

P : Premium for each year

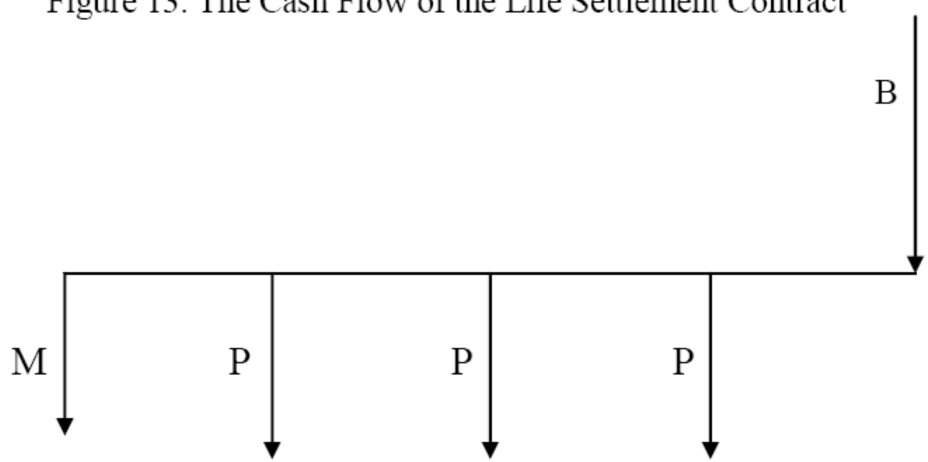
i : Yield to maturity

V : Discount rate, $V = \frac{1}{1+i}$, i is the yield to maturity

M : Purchase Price for the life settlement

T : Random variable of life time

Figure 13. The Cash Flow of the Life Settlement Contract



Pricing

The formula for calculating the Purchase Price M at the purchase date is:

$$M = \mathrm{E}[BV^T - P\left(\frac{1 - V^T}{iV}\right)]$$

Here T is the (random) life length, and $BV^T = \frac{B}{(1+i)^T}$ measures the present value of the net death benefit. This is the positive cash flow for the life settlement purchaser.

$P\left(\frac{1-V^T}{iV}\right) = \sum_{t=1}^{T-1} \frac{P}{(1+i)^t}$ measures the net present value of the premiums P which the purchaser has to continue to pay until the death of the policy holder. This is the negative cash flow for the life settlement purchaser.

Table 15. Life Settlement Prices for the Different Yield to Maturity

Yield to Maturity ***i***	**Life Settlement Price** **M ($)**
5.00%	41190.75
6.00%	40071.93
7.00%	39010.62
8.00%	38002.51
9.00%	37043.73
10.00%	36130.74
11.00%	35260.35
12.00%	34429.67
13.00%	33636.03
14.00%	32877.02
15.00%	32150.44
16.00%	31454.24
17.00%	30786.58
18.00%	30145.72

Another Example

Male has estimated life expectancy of 8 years, US life table has expectation 28.5 years

ADJUSTED AND STANDARD MORTALITY TABLE FOR MALE AGE 45.

YEAR k	AGE x	STANDARD TABLE RATE q_x	STANDARD TABLE PROBABILITY g_k	STANDARD TABLE SURVIVAL FUNCTION l_k	ADJUSTED TABLE RATE q'_x	ADJUSTED TABLE PROBABILITY f_k	ADJUSTED TABLE SURVIVAL FUNCTION l'_k
0	45	0.00444	0.00444	1.00000	0.09667	0.09667	1.00000
1	46	0.00493	0.00491	0.99556	0.09840	0.08888	0.90333
2	47	0.00548	0.00543	0.99065	0.10041	0.08178	0.81445
3	48	0.00608	0.00599	0.98522	0.10244	0.07506	0.73267
4	49	0.00673	0.00659	0.97923	0.10444	0.06868	0.65762
5	50	0.00746	0.00726	0.97264	0.10681	0.06290	0.58893
6	51	0.00825	0.00796	0.96539	0.10918	0.05743	0.52603
7	52	0.00905	0.00866	0.95742	0.11091	0.05197	0.46860
8	53	0.00985	0.00935	0.94876	0.11191	0.04662	0.41663
9	54	0.01068	0.01003	0.93941	0.11253	0.04164	0.37000
10	55	0.01153	0.01072	0.92938	0.11265	0.03699	0.32837
11	56	0.01249	0.01147	0.91866	0.11307	0.03295	0.29138
12	57	0.01370	0.01243	0.90719	0.11486	0.02968	0.25843
13	58	0.01525	0.01365	0.89476	0.11850	0.02711	0.22875
14	59	0.01706	0.01503	0.88112	0.12319	0.02484	0.20164
15	60	0.01907	0.01652	0.86608	0.12840	0.02270	0.17680
16	61	0.02113	0.01795	0.84957	0.13318	0.02052	0.15410
17	62	0.02315	0.01925	0.83162	0.13706	0.01831	0.13358
18	63	0.02504	0.02034	0.81236	0.13959	0.01609	0.11527
19	64	0.02688	0.02129	0.79202	0.14124	0.01401	0.09918

Other types of information incorporated

Consider a person who has experienced a life event that has changed their medical information. For example a spinal cord injury . We know relative mortality impact. How do we adjust life table?

Cumulative Seven Year Relative Mortality Ratios for Spinal Cord Injury Patients

	Age at Time of Injury		
Neurologic Level	1–24	25–49	50+
Incomplete Paraplegia	4.82	6.59	3.26
Complete Paraplegia	4.93	6.93	3.26
Incomplete Quadriplegia	4.22	6.71	3.95
Complete Quadriplegia	12.40	20.78	14.11

Source: DeVivo, M.J., et al. "Seven-Year Survival Following Spinal Cord Injury," *Arch Neurol*, 44 (1987): 872–875.

- We assume the level of injury (ip,cp,iq,cq) influences mortality rate relative to standard by increasing additively, e.g for incomplete paraplegics, mortality is $= \mu_x + \mu_{ip.}$ The question is how to estimate such that the adjusted mortality table has the 7 year mortality ratios as given in the previous table.

$$\mu_x' = \begin{cases} \mu_x + \mu_{ip} & \text{for incomplete paraplegia,} \\ \mu_x + \mu_{cp} & \text{for complete paraplegia,} \\ \mu_x + \mu_{iq} & \text{for incomplete quadriplegia, and} \\ \mu_x + \mu_{cq} & \text{for complete quadriplegia,} \end{cases}$$

We also need the number of exposure units E_x in each age group (to get average age at death)const

$$\sum_{x=1}^{110} E_x(1 - {}_7p_x') = f_k \sum_{x=1}^{110} E_x(1 - {}_7p_x)$$

Getting a new adjusted mortality Table

$$\sum_{x=1}^{110} E_x(1 - {}_7p'_x) = f_k \sum_{x=1}^{110} E_x(1 - {}_7p_x)$$

Age at death constraint: same for observed and predicted

Solving these we obtain

$$\mu_k = -\frac{1}{7} \ln \left[\frac{\sum_{x=1}^{110} E_x(1 - f_k(1 - \exp(-\sum_{j=0}^{6} \mu_{x+j})))}{\sum_{x=1}^{110} E_x \exp(-\sum_{j=0}^{6} \mu_{x+j})} \right] \qquad (3)$$

Values Found Using Equation (3)

	Value
μ_{ip}	0.013826
μ_{cp}	0.014248
μ_{iq}	0.015494
μ_{cq}	0.074946

We want to use information theory to incorporate the previous development. Find 4 series of mortality rates $\delta_{x,k}$ for lesion type k. We chose $\delta_{x,k}$ to minimize the distance from $\mu_{x,k}$. The derived $\delta_{x,k}$ should satisfy constraints:

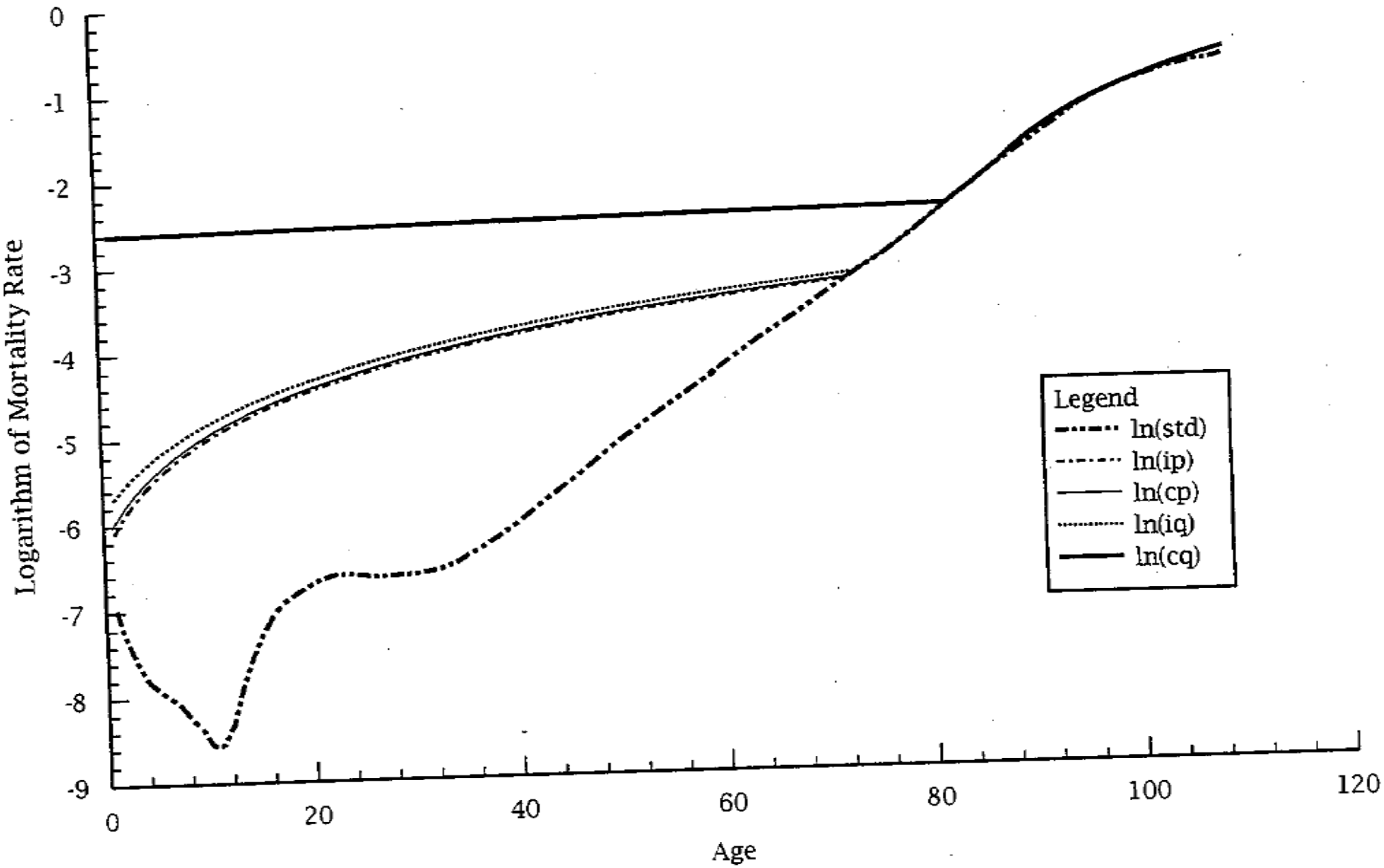
- Increase with age;
- Be convex;
- Be smooth;
- Have the number of deaths in the adjusted table using δ_x match the number of deaths found empirically by the DeVivo et al. (1987) study;
- Have an ordered relationship between different neurologic severity levels; and
- Be nonnegative.

Accordingly, the modified mortality table is obtained as the solution to:

$$\min I(\Delta|\mu) = \sum_{k} \sum_{x=1}^{110} \delta_{x,k} \ln(\delta_{x,k}/\mu_x) \quad (5)$$

Figure 1

Adjusted Mortality Using Information Theory



Thank you !